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Luminosity Limitations for Colliders Based on Plasma Acceleration

Valeri Lebedev & Sergei Nagaitsev

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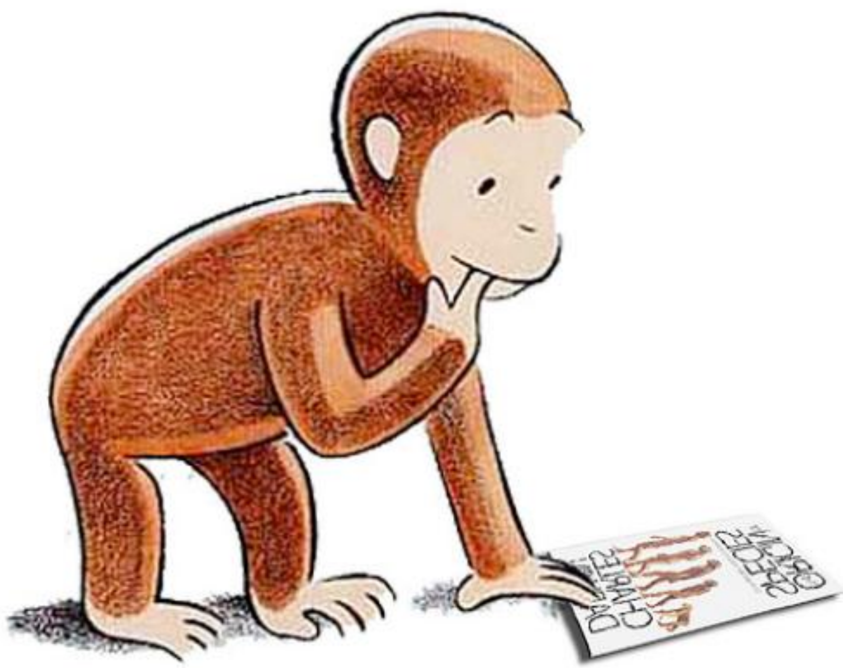


Accelerator Physics & Technology Seminar

Fermilab

April 30, 2013

Objective

- An active discussion on the R&D for plasma accelerators in application to e+e- colliders in the TeV energy range
  - Two competing proposals:
    - beam-based from SLAC/UCLA
    - and laser-based from LBNL
- Figure of merit
  - Luminosity $$L = \frac{fN^2}{4\pi\sigma_x\sigma_y} = \frac{P_{beam}}{4\pi E_b}\frac{N}{\sigma_x\sigma_y}$$
  - $N/\sigma_x\sigma_y$ is limited by disruption and beamstrahlung
  - Energy efficiency (P_{beam}/P_{total}) is the primary issue

- Our main goal is to discuss the collider applications of plasma-based technology.
  - We have found the plasma acceleration to be an interesting research topic. The technology may have many applications!

Introduction

- A design for a conventional e+e- collider of up to 3 TeV exists! A plasma-based concept is being offered to the HEP community as a cost-saving proposal only (or as an after-burner).
  - The proponents argue that plasma-based colliders could cost less because they could be made shorter (in overall length) and with fewer components;
  - We know from existing cost estimates that the overall length is just one of many factors going into the facility cost. Others include complexity, total power, risk of new untested technologies, R&D costs, etc. We will not discuss this in our talk but we should keep it in mind;
  - The proponents differ in their opinions on how long it will take to develop the collider technology (20 - to more than 50 years). The R&D in the US is funded at the level of ~20M$/year and the proponents argue for more funding.

Introduction (cont'd)

- To compete with ILC or CLIC designs, a plasma-based concept needs to achieve a luminosity of ~2×10^{34} at ~1 TeV c.m.

- The upper energy for an electron-positron collider ~3 TeV is limited by beamstrahlung (not by accelerating technology).

- We should keep in mind that the HEP community is asking for an electron-positron collider, maybe gamma-gamma, NOT electron-electron. Thus, it is important for a plasma-based concept to work equally well for both electrons and positrons.

Outline

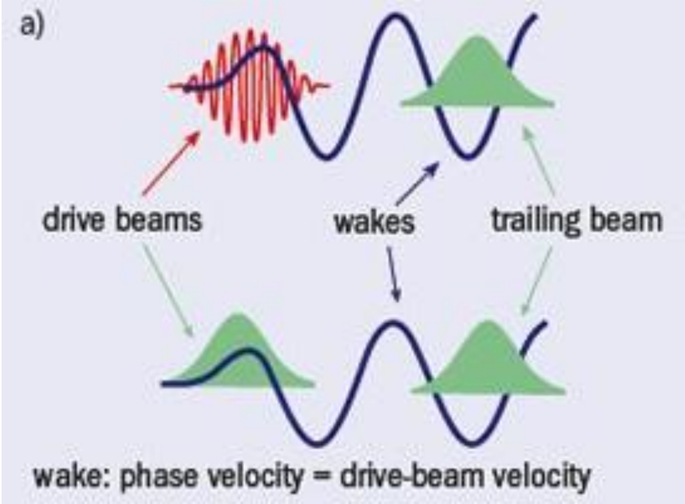
- Short history of plasma acceleration and major achievements
- Basics of plasma acceleration
- Luminosity limitations for plasma based colliders
- SLAC and LBNL proposals
- Conclusions

Huge community resonance…

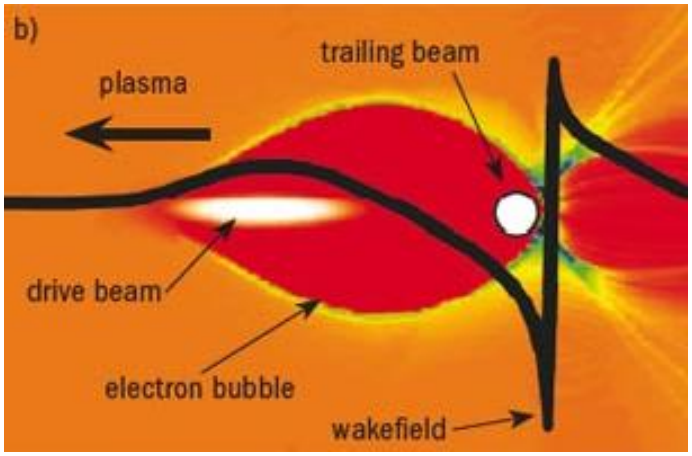
- Large number of enthusiastic supporters
- Presented as extremely promising for TeV range e+e- colliders

Short History of Plasma Acceleration

- Two ways to create a plasma wave
  - Laser-driven wake-field accel.: Tajima and Dawson, 1979
  - Beam-driven wake-field accel.: Chen, Dawson, et al., 1985
- Two regimes
  - Quasi-linear
  - Bubble (or blowout)
- Tremendous progress at both fronts
  - Laser excited plasma – LBNL
    - 30 GeV/m, 3 cm, 1 GeV
  - Electron bunch excited plasma – SLAC
    - ~50 GeV/m, ~0.85 m, ~40 GeV



Linear wake

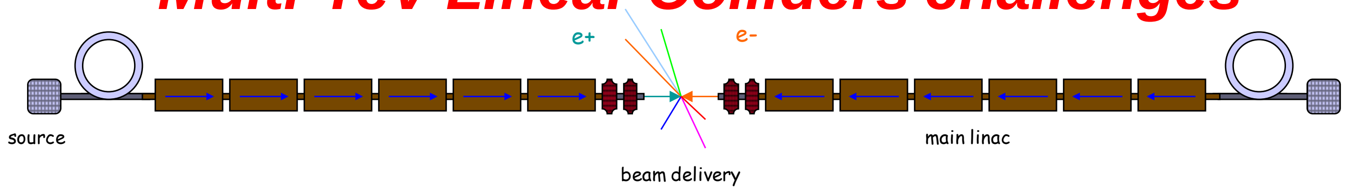


Nonlinear wake

Main issue for collider applications:

- How to make plasma acceleration efficient (in terms of power transfer to the beam),
  - while maintaining beam parameters suitable for a collider application (small emittance and energy spread)

Multi-TeV Linear Colliders challenges



Energy reach $$E_{cm} = 2\,F_{fill}\,L_{linac}\,G_{RF}$$

Luminosity $$L = \frac{n_b N^2 f_{rep}}{4\pi\sigma_x^*\sigma_y^*} \times H_D \propto \frac{\eta_{beam}^{AC}\,P_{AC}}{\varepsilon_y^{1/2}}\,\frac{\delta_{BS}^{1/2}}{E_{cm}}$$

Limitation by practicalities:

Wall plug power: mitigation power to beam transfer efficiency

Wall plug power <300MW @ 3TeV,
$L_{0.01} = 2.10^{34}$ ⇨ 20 MW/beam ⇨ **Wall plug to beam efficiency > 13%**

Cost : mitigation by high accelerating gradient

Total extension < 10km @ 3TeV
Each linac < 2.5 km ⇨ **Effective Accelerating Gradient ~ 1 GV/m**

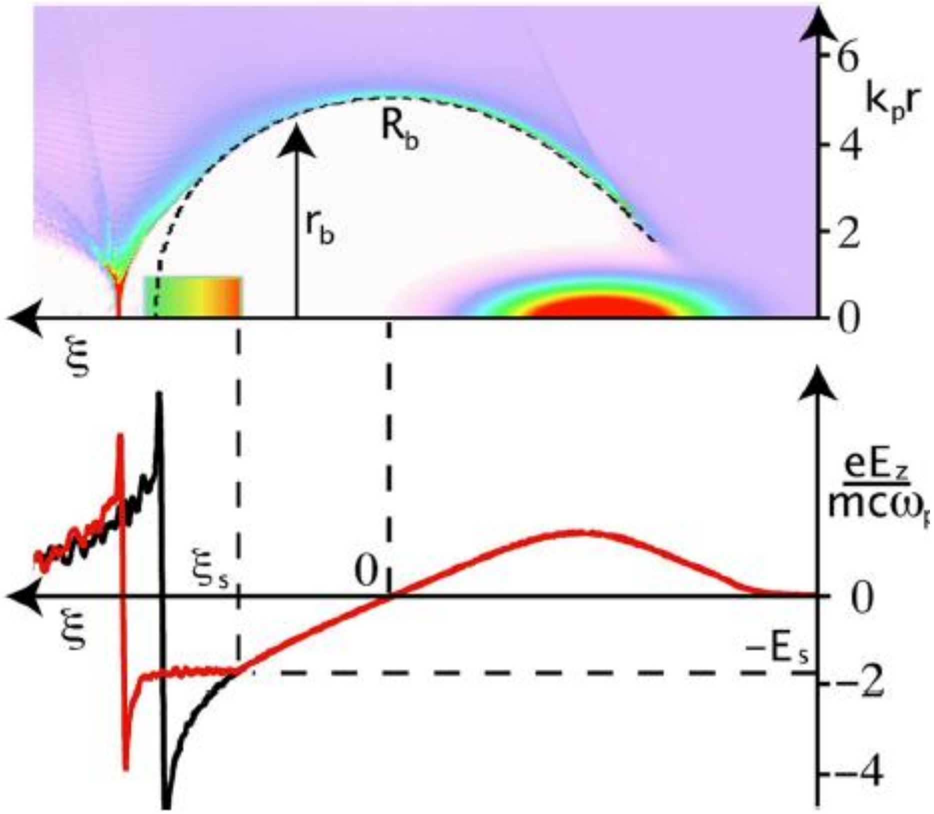
J.P.Delahaye @ MIT April 11,2013

Acceleration in ILC cavities

- The ILC cavity: ~1 m long, 30 MeV energy gain; f_0 = 1.3 GHz, wave length ≈ 23 cm
- The ILC beam: 3.2 nC (2×10^{10}), 0.3 mm long (rms); bunches spaced ~300 ns (90 m) apart
- Each bunch lowers the cavity gradient by ~15 kV/m; this voltage is restored by an external rf power source (Klystron) between bunches
- Such operation of a conventional cavity is only possible because the Q-factor is >> 1; the RF energy is mostly transferred to the beam NOT to cavity walls.

Acceleration in plasma

- The Q-factor is very low (for high fields) – must accelerate the bunch within one plasma wavelength of the driver!
- Cannot add energy between bunches, thus a single bunch must absorb as much energy as possible from the wake field.



M. Tzoufras et al., PRL 101, 145002 (2008)

Beam-driven Wake: Experimental Results

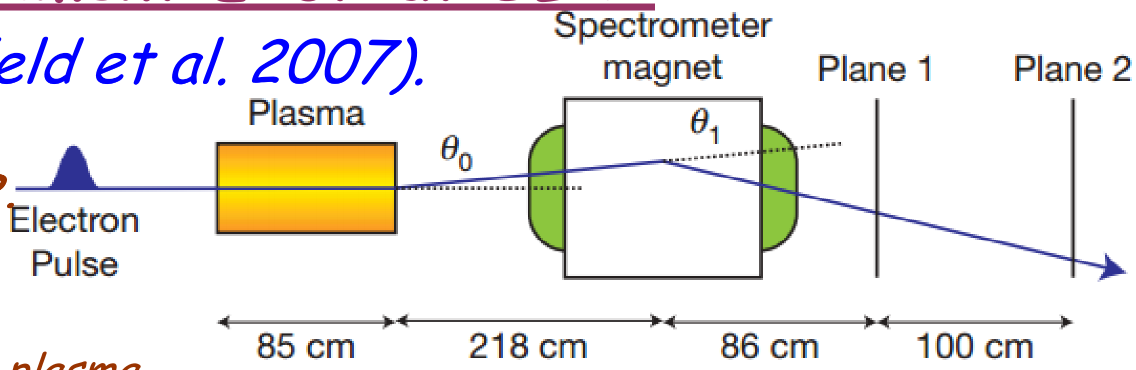
The UCLA/USC/SLAC experiment E167 at SLAC

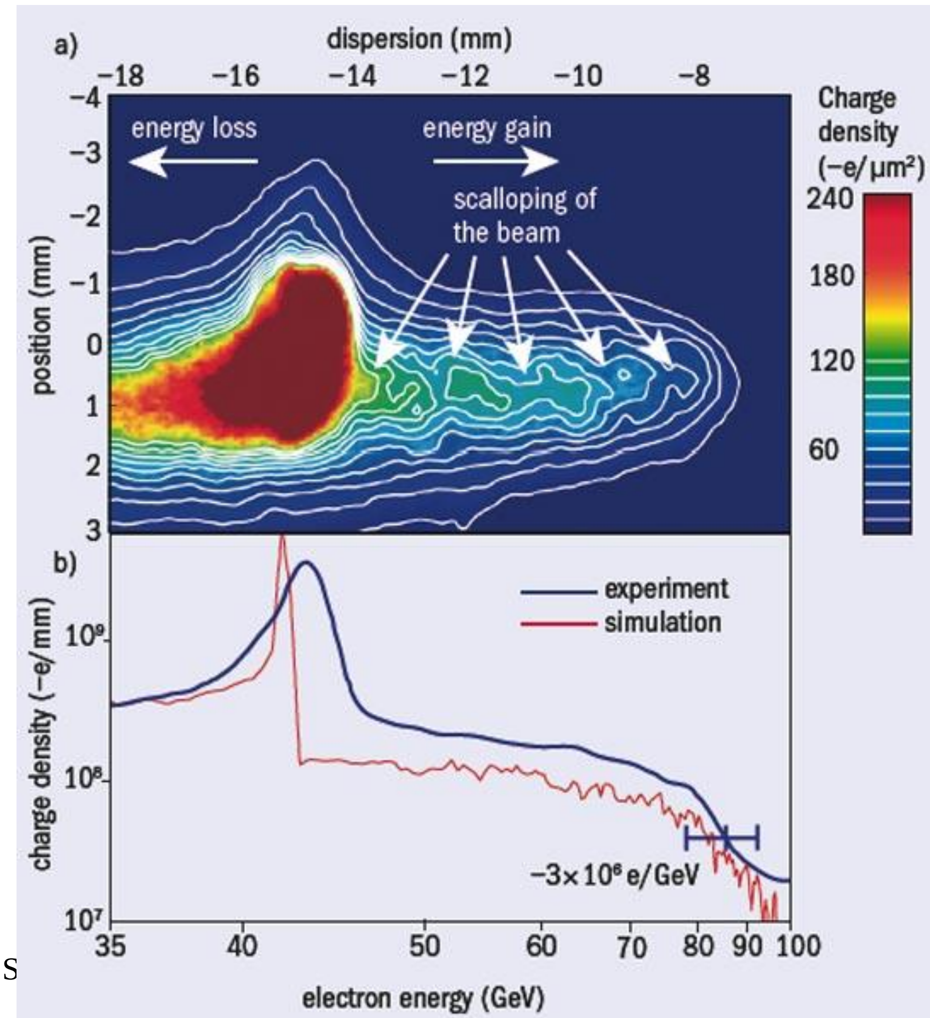
- (SLAC-PUB-12363, *Blumenfeld et al. 2007*).

Energy spectrum of the electrons in the 35–100 GeV range as observed in plane 2.

a) Head of the pulse is unaffected by plasma and is at 43 GeV.
Core of the pulse lost energy driving the plasma wake and is partly out of camera field of view.
Tail particles reached energies up to 85 GeV.
Pulse envelope exits the plasma with an energy-dependent betatron phase advance which is consistent with observed scalloping of the beam.

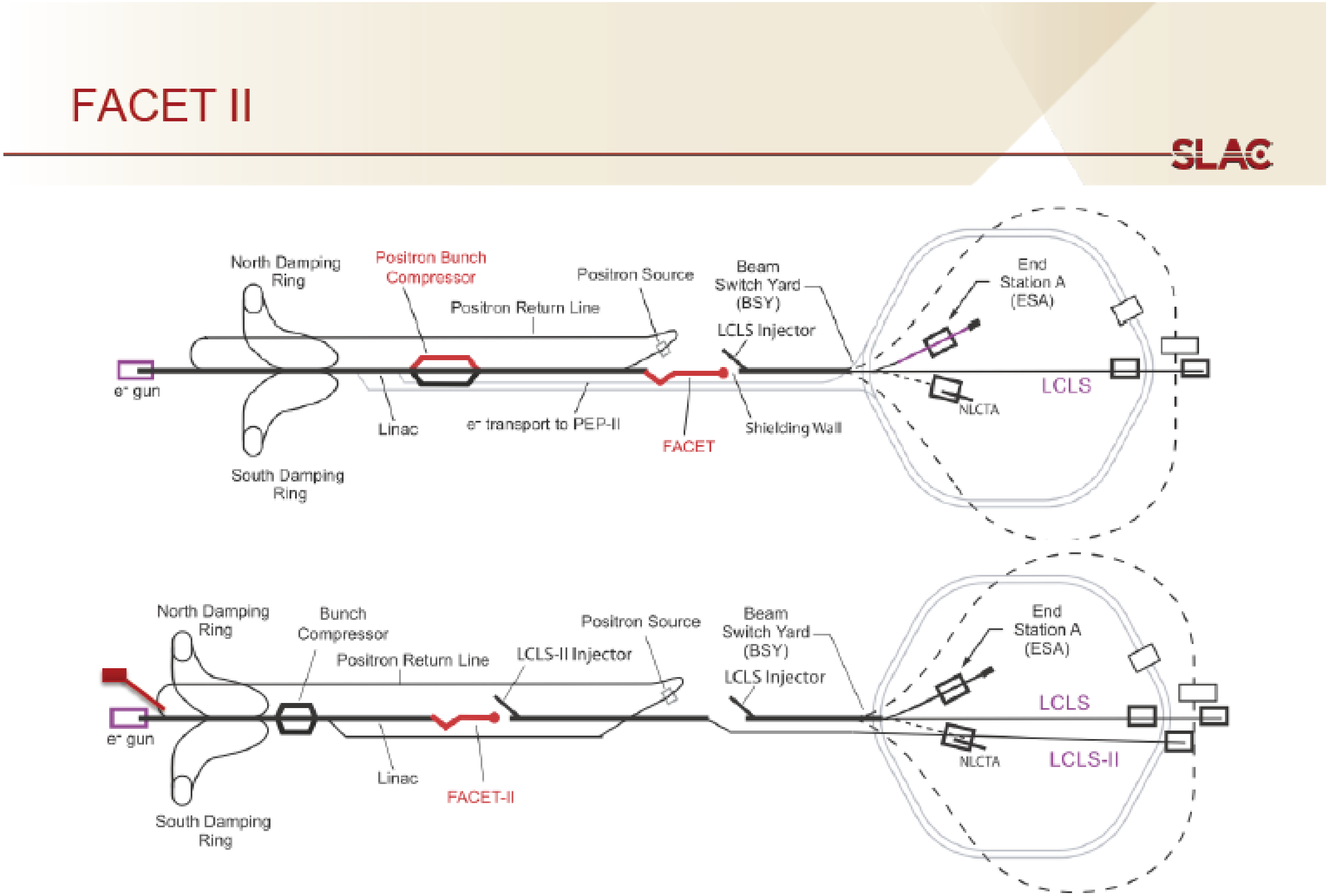
b) Projection of the image in a, shown in blue. The simulated energy spectrum is shown in red.





- Plasma density (Li), $n_e = 2.7\cdot10^{17}$ cm^{-3}
- Initial energy – 42 GeV
- Final energy ~85 GeV
- Accelerating gradient ~50 GeV/m
  - More than 3 orders of magnitude larger than ILC

FACET (existing) and FACET-II (proposed) at SLAC

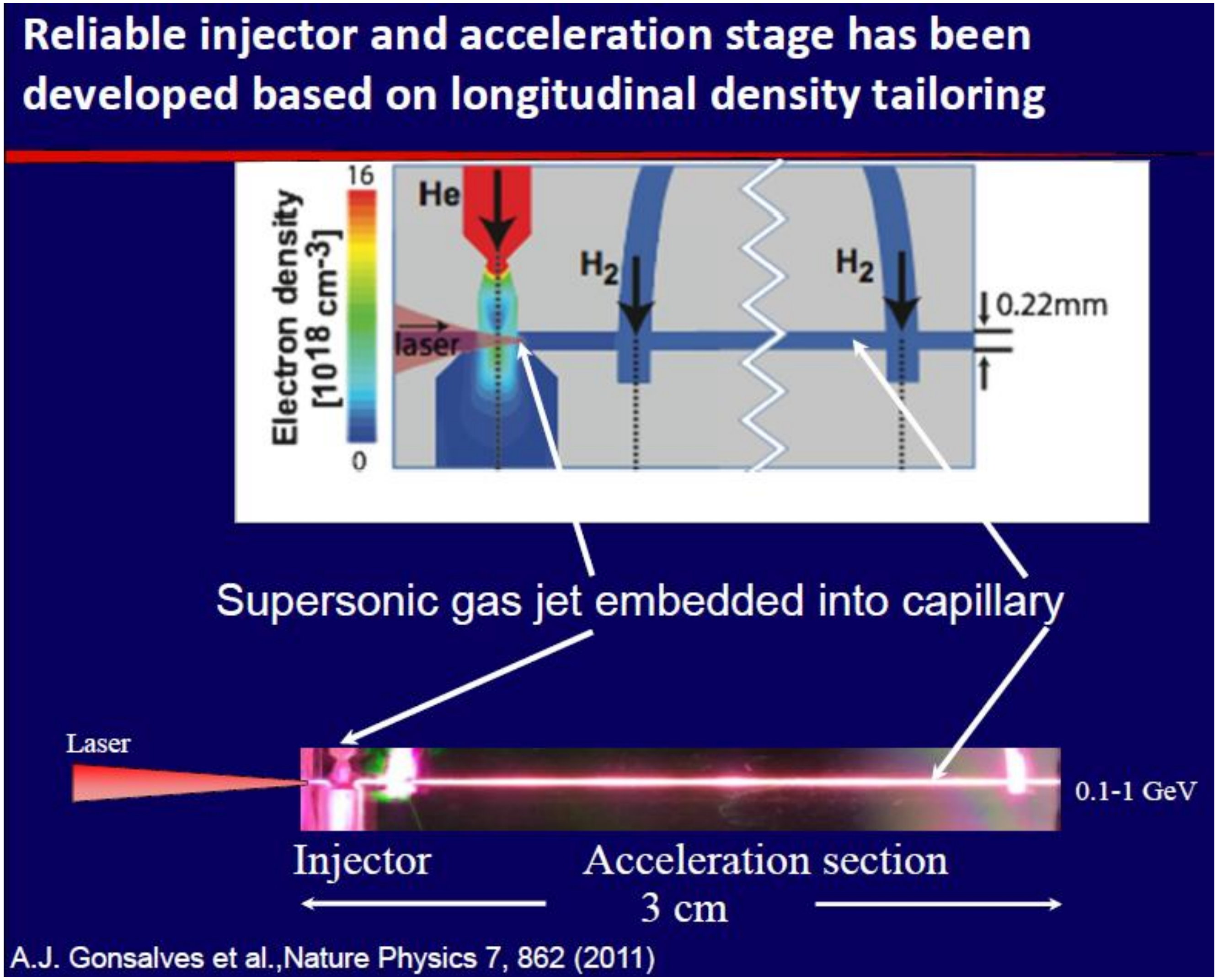


In early 2016, LCLS-II, will begin commissioning using part of the tunnel occupied by FACET

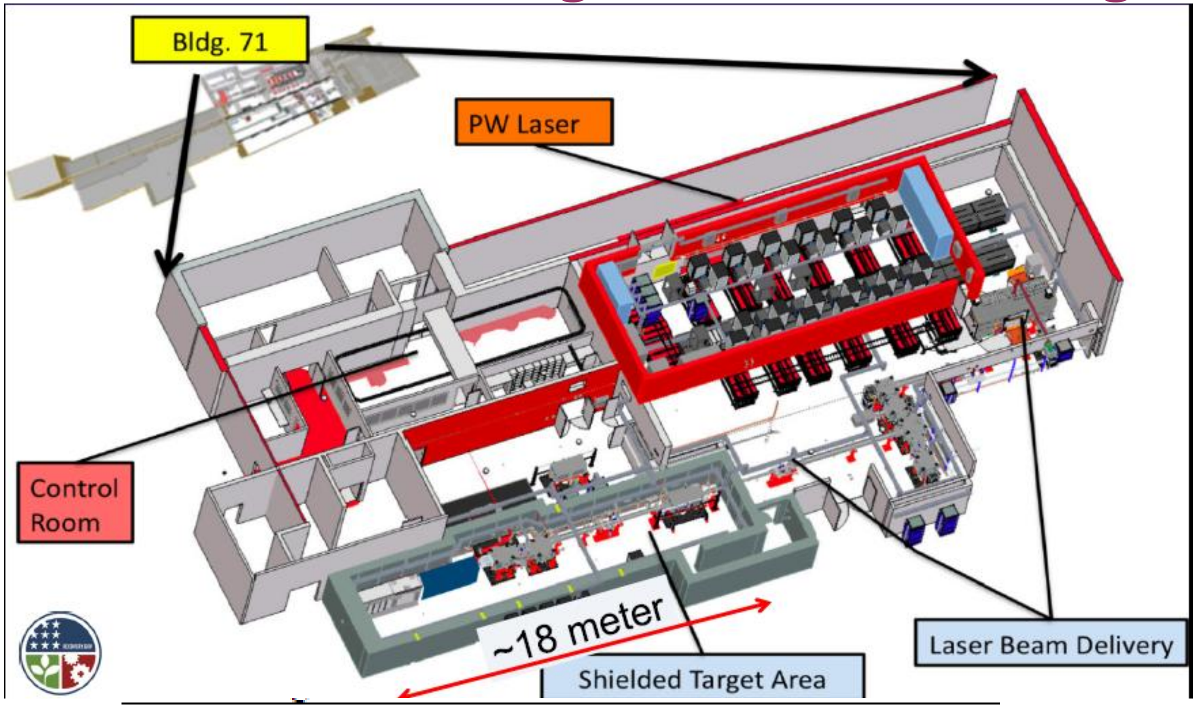
- Facet-I goal: to double the energy of a witness bunch (25->50 GeV) with a narrow energy spread but with a high efficiency

Laser-driven Wake: Experimental Results

- Several active groups around the world
  - ~1 GeV electron energy achieved at LBNL



BELLA (LBNL) aims at achieving 10 GeV in a single stage



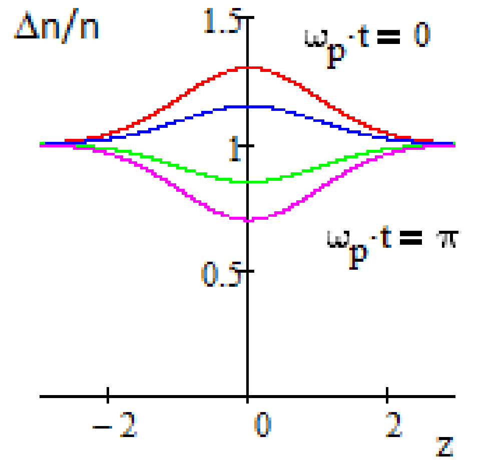
Laser & Plasma Parameters	BELLA 10 GeV
Laser energy [J]	40
a_0	1.4
λ_p [μm]	107.5
$k_p L_{laser}$	1
L_{laser} [fs]	57
w_0 [μm]	91.4
P [TW]	554
$k_p w_0$	5.3
P/P_c	1.7
Linear dephasing length [m]	0.97
Pump depletion length [m]	1.98
Stage length in simulation [m]	0.6
Energy gain in simulation [GeV]	10

Plasma Waves and Acceleration in Plasma

- **Plasma frequency**

  ω_p is the only frequency present in linear regime:

$$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

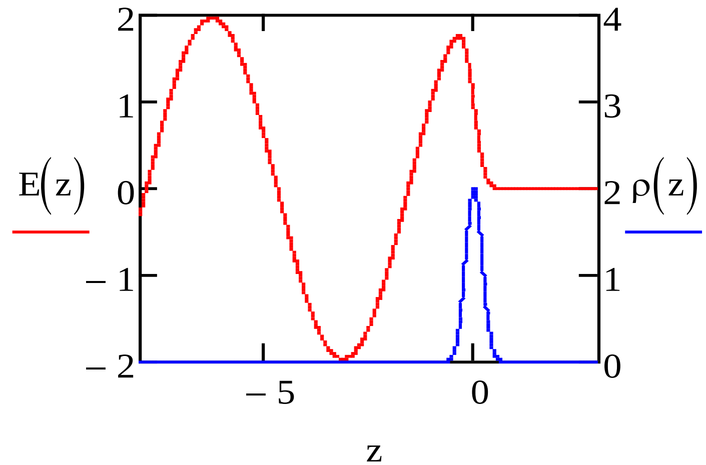


- **Plasma wave-vector**

  Reaction of plasma to a short pulse excitation determines the period of oscillations: $kv = \omega_p \,, \quad \lambda = 2\pi / k$

  => for an ultra-relativistic bunch:

$$k_p = \omega_p / c$$

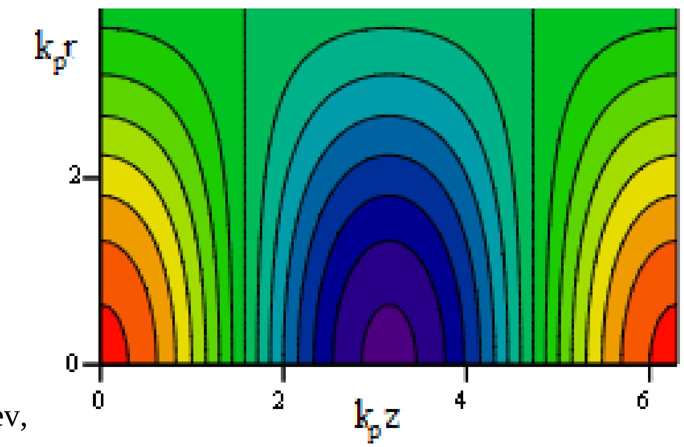


- Maximum electric field

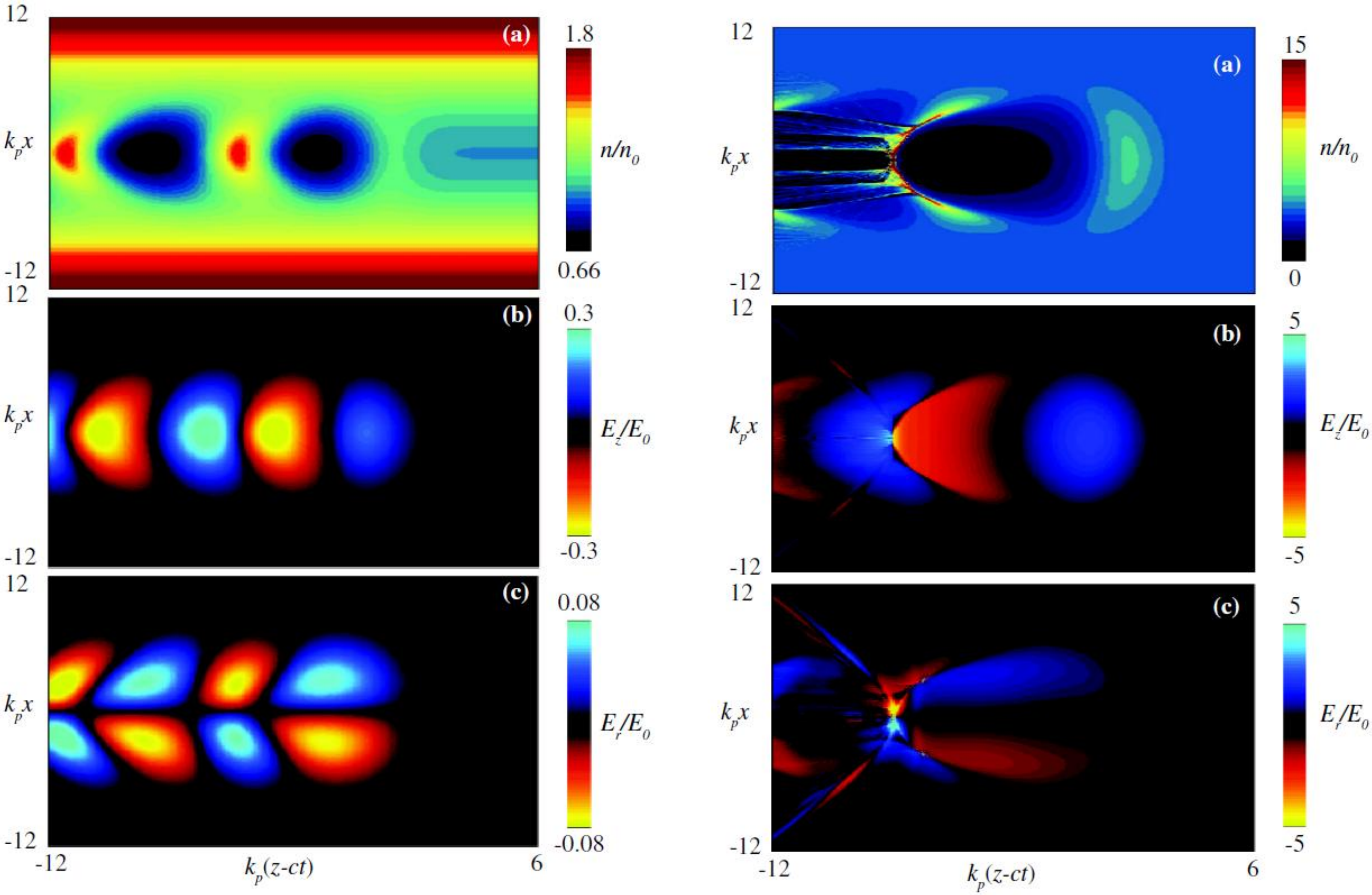
$\mathrm{div}(\mathbf{E}) = 4\pi\rho$ => for plane plasma wave

$$\frac{dE_z}{dz} = 4\pi e n \cos k_z z \quad \Rightarrow \quad E_z = \frac{4\pi e n}{k_z} \sin(k_z z)$$

$$E_{\max} = \frac{4\pi e n_e}{k_p} = 30.2 \frac{\mathrm{GV}}{\mathrm{m}} \sqrt{\frac{n_e}{10^{17}\,\mathrm{cm}^{-3}}}$$



Regimes of Acceleration in Plasma



Linear regime, $\Delta n/n \approx 0.3$ *Blowout regime: $\Delta n/n=1$*

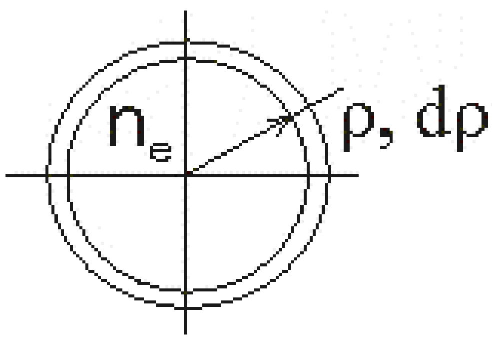
*Pictures are from [1]: "Physics considerations for laser-plasma linear colliders", PRST-AB, **13**, 101301 (2010)*

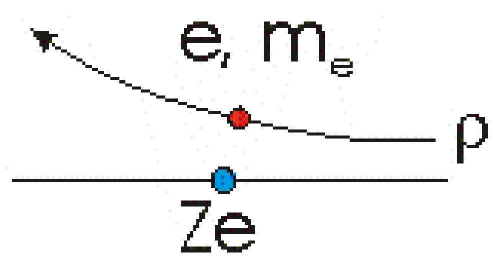
Single Particle Deceleration in Plasma

- The theory is well-known (details can be found in plasma text books)
- A two step solution:
  1. **Collective plasma response**
     - Solution of Maxwell equations with $\varepsilon = 1 - \omega_p{}^2 / \omega^2$
     - Solution diverges at small impact parameters where perturbation theory does not describe particle motion in plasma
  2. **Particle interaction with independent plasma electrons**
     - Energy transfer to a single electron => Deceleration rate

$$\Delta p_\perp = \frac{Ze^2}{\rho^2}\frac{2\rho}{\mathrm{v}}, \quad \Delta \mathrm{E} = \frac{\Delta p_\perp{}^2}{2m_e}, \quad \frac{d\mathrm{E}}{dx} = \int \Delta \mathrm{E}\, n_e 2\pi\rho d\rho = \frac{4\pi n_e Z^2 e^4}{m\mathrm{v}^2}\int_0^\infty \frac{d\rho}{\rho}$$

     - Solution diverges at large impact parameters where screening of particle field by plasma need to be taken into account





Single Particle Deceleration in Plasma (continue)

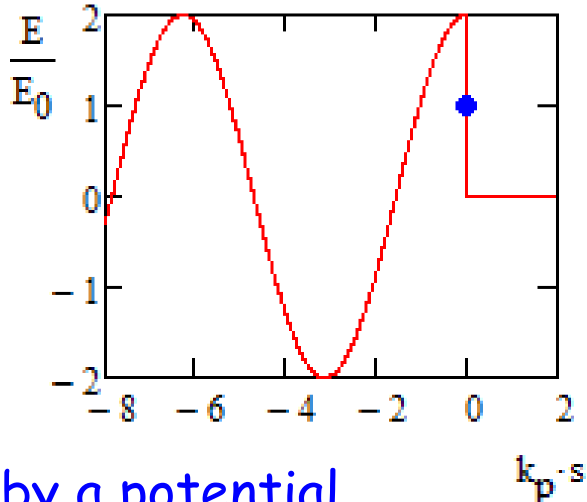
- Accurate combination of two approaches yields for heavy particle ($m \gg m_e$)

$$\boxed{\left(\frac{dE}{ds}\right)_0 \approx \frac{4\pi n_e Z^2 e^4}{m_e v^2}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right),} \qquad \rho_{\max} \approx 1.123\frac{v}{\omega_p}, \quad \rho_{\min} = \frac{Ze^2}{m_e v^2}, \quad \frac{2}{e^{\gamma}} \approx 1.123, \quad \gamma \approx 0.577 .$$

  - ♦ It works well as long as $\rho_{\max} \gg \rho_{\min}$ and the theory of classic scattering is applicable - $v/c \ll Z\left(e^2/\hbar c\right) \approx Z/137$.
  - ♦ For relativistic multi-charged ion $\ln\left(\rho_{\max}/\rho_{\min}\right) \approx 14$.

- For a single particle the longitudinal wake (longitudinal field at distance *s* after the particle) is

$$\frac{dE}{ds} = \left(\frac{dE}{ds}\right)_0 2\cos\left(k_p s\right)$$



- The solution also yields that B=0 in the wake field trailing the ultra-relativistic particle
  => the wake field behind the particle can be described by a potential

$$\boxed{\varphi(\mathbf{r},t) = 2Zek_p \mathrm{K}_0\left(\rho k_p\right)\sin\left(k_p s\right), \quad s = z - ct < 0, \quad \rho > \rho_{\min}}$$

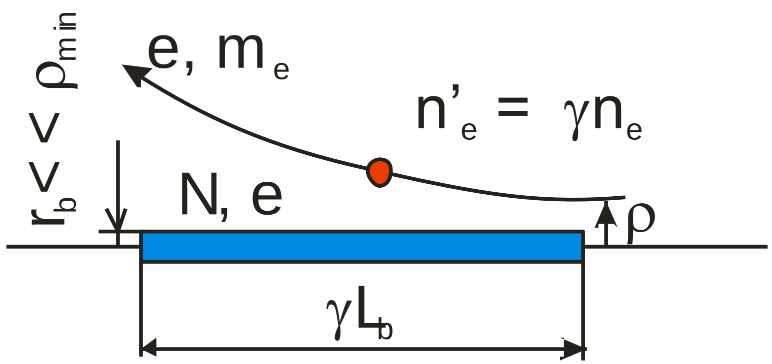
For $\rho \ll \rho_{\max} \approx k_p^{-1}$ that yields the potential of charged wire

$$\varphi(\mathbf{r},t) \approx 2Zek_p \sin\left(k_p s\right)\left(\ln\left(k_p \rho\right) + const\right)$$

Calculations also reveal that el. density is not perturbed ($n_e = n_i$) for $\rho > \rho_{\min}$.

Deceleration of an Ultra-relativistic Bunch

- For a sufficiently small number of particles the bunch deceleration is not much different from single particle
  - Field screening and ρ_{max} will stay the same
  - ρ_{min} will be modified because of finite bunch length
  - Deceleration will be growing from bunch head to its tail
- Consideration of collisions at small impact parameters looks much simpler in the beam frame
  - where $L_b \gg \rho_{max}$ and simple formulas for cylindrically symmetrical case can be used for trajectory calculations



$$E_\rho = 2\delta q / \rho$$

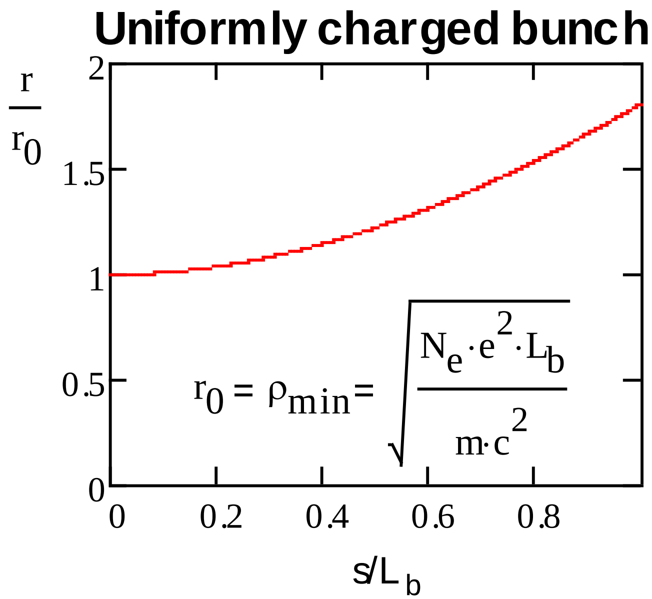
δq is the total plasma related charge inside unit length cylinder with radius ρ

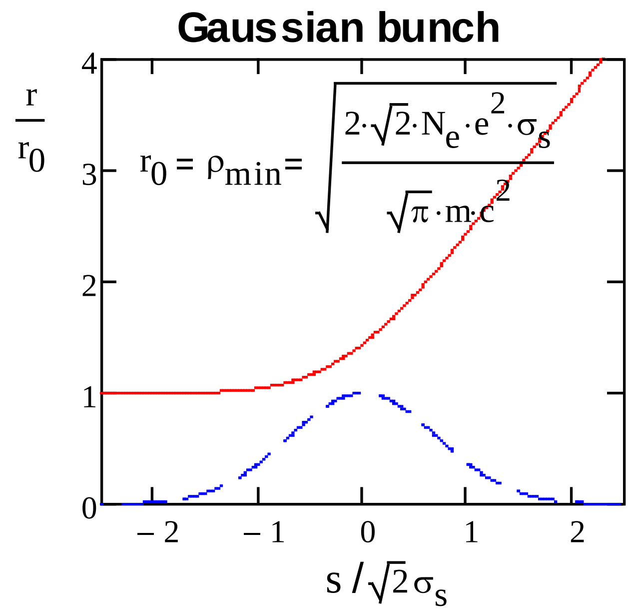
We assume zero ⊥ beam size (actually any size < ρ_{min} is small enough)

  - Solving equations of motion determines $\rho(s,\rho_0)$
  - Integration over impact parameters yields decelerating field in b.f.

$$\delta\varphi'(s) = 2\delta Q' \ln\frac{\rho(s',\rho_0)}{\rho_0} \quad \Rightarrow \quad eE' = -e\frac{d\varphi'}{ds'} = 4\pi n'_e e^4 \frac{d}{ds'}\left(\int_0^{\rho_{max}} \rho \ln\left(\frac{r(s',\rho)}{\rho}\right) d\rho\right)$$

Case of Electron Bunch





Plasma electron scattering on the bunch with impact parameter ρ_{min}

- Trajectory shape ρ/ρ_{min} depends only on ρ_0/ρ_{min}
  - Definition of ρ_{min}: $\delta\rho \equiv (\rho - \rho_0) \approx \rho_0$ for $\rho_0 = \rho_{min}$
- Integration yields contribution of small impact parameters
- Combining it with contribution of large impact parameters determines the deceleration force along the bunch

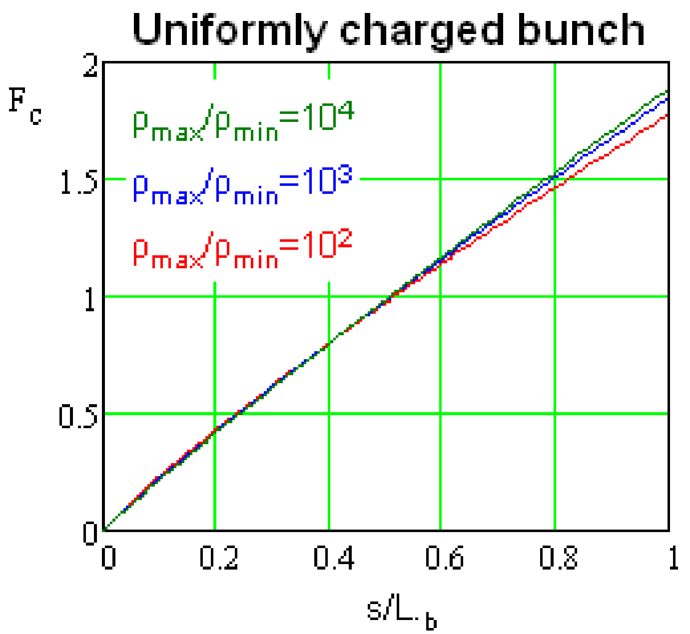
Case of Electron Bunch (continue)

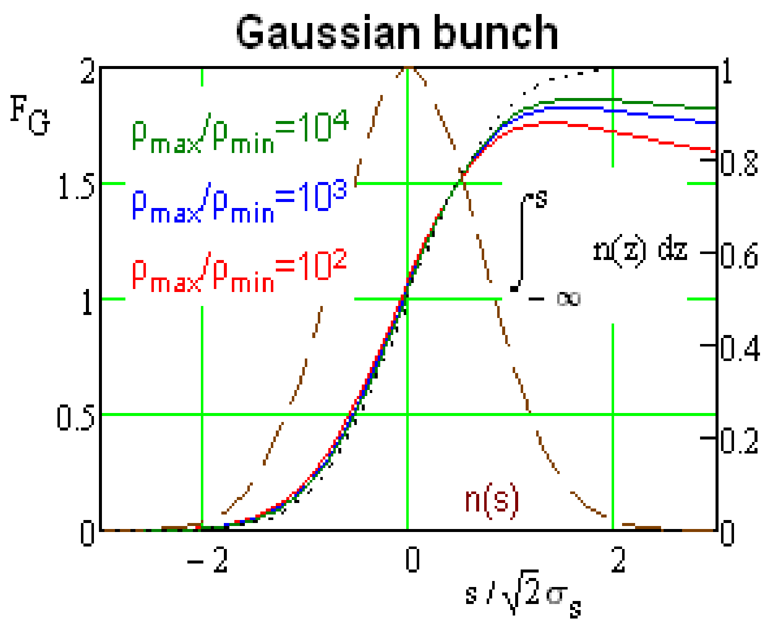
- For uniformly charged and sufficiently short ($L_b \ll 2\pi/k_p$) bunch

$$\frac{dE}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} \ln\left(\frac{1.3\rho_{\max}}{\rho_{\min}}\right) F_c\left(\frac{\rho_{\max}}{\rho_{\min}}, \frac{s}{L_b}\right), \quad F_c(X,s) \approx \frac{1}{\ln(1.3X)} \int_0^X \frac{(2\rho + 0.23s)s\rho\, d\rho}{\left(\rho^2 + s(1+0.23\rho)\right)(\rho + 0.23s)}$$

- For Gaussian bunch($\sigma_s \ll 1/k_p$)

$$\frac{dE}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) F_G\left(\frac{\rho_{\max}}{\rho_{\min}}, \frac{s}{\sqrt{2}\sigma_s}\right), \quad F_G(X,s) \approx \frac{2 - \operatorname{erfc}(s)}{2\ln X} \ln\left(\frac{2X^2}{\sqrt{\pi}s\left(2 - \operatorname{erfc}(s)\right) + \exp\left(-s^2\right)}\right)$$

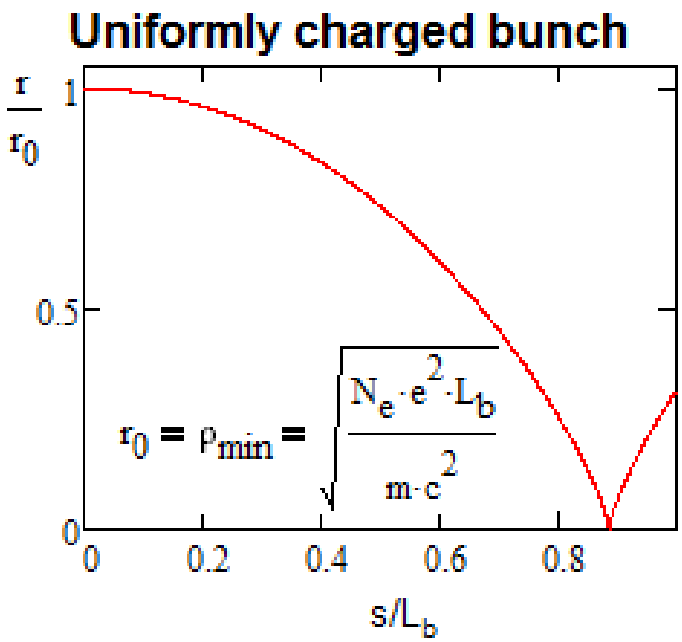


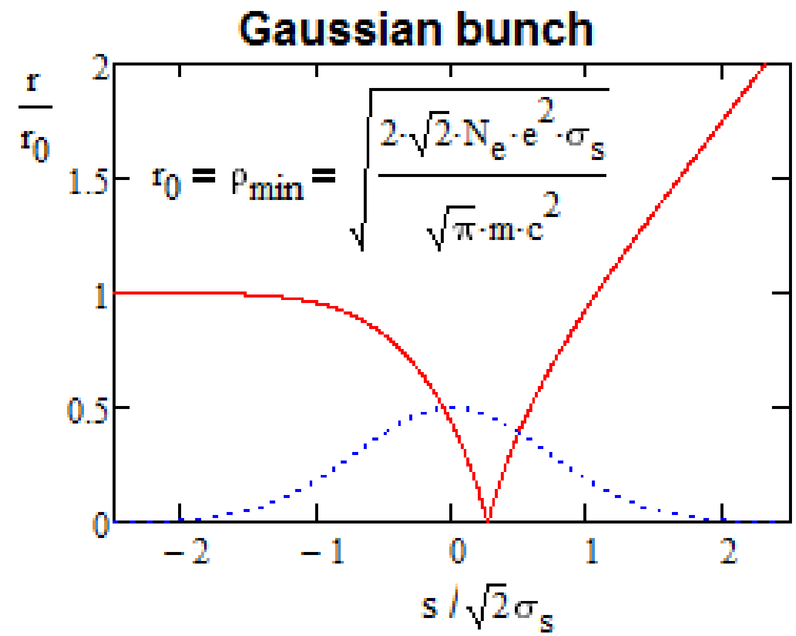


$$\operatorname{erfc}(s) = \frac{2}{\sqrt{\pi}} \int_s^\infty e^{-x^2} dx$$

- The longitudinal wake is close to a step function: $W(s) \propto \theta(s)\cos(k_p s)$
  - strictly speaking it is dependent on the longitudinal density distribution in the bunch but it makes only logarithmic correction

Case of Positron Bunch



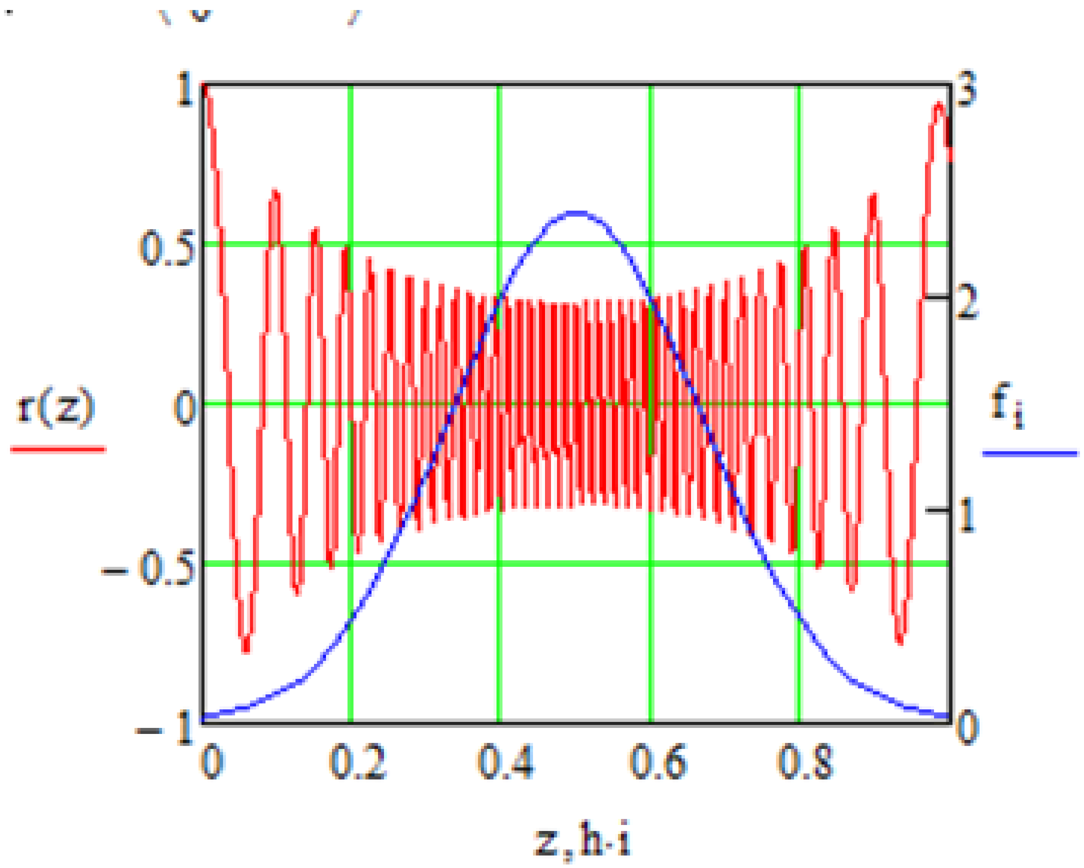


Plasma electron scattering on the bunch with impact parameter ρ_{min}

- For $\rho < \rho_{min}$ the field of positron bunch pinches plasma electrons
  - It slightly increases an interaction with plasma and the deceleration force
  - It also makes Coulomb logarithm dependent on s
- For uniformly charged positron bunch ($L_b \ll 2\pi/k_p$)

$$\frac{dE}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2}\frac{2s}{L_b}\ln\left(\frac{1.4\rho_{max}}{\rho_{min}}\frac{L_b}{s}\right), \quad s \in [0, L_b], \quad \rho_{max} \approx 1.123c/\omega_p$$

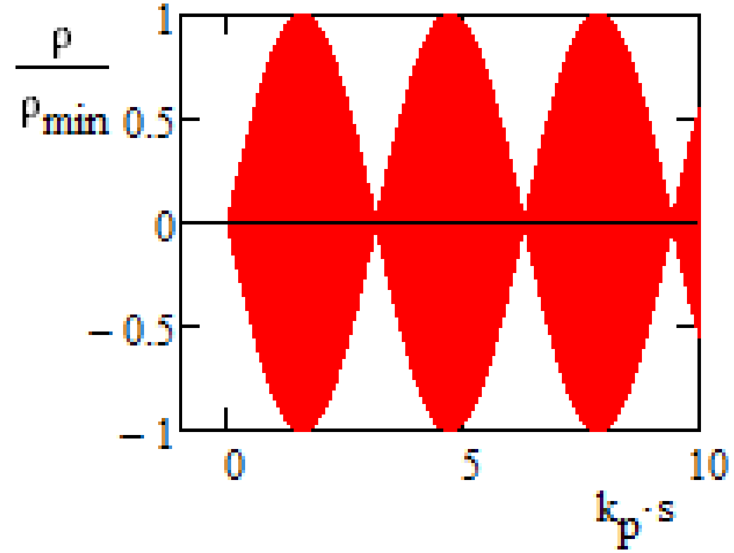
Collapse of electrons inside of the positron bunch



A trajectory of a plasma electron inside of the positron bunch (4×10^9).

Validity Limits of Approximation

- Longitudinal wake function is obtained in a logarithmic approximation
  - It is justified only for $\rho_{min} \ll \rho_{max}$
- If bunch radius is larger than ρ_{min} but is still much smaller than ρ_{max} it should be used instead of ρ_{min}
- Thin electron bunch ($\sigma_{\perp n} \le \rho_{min}$) leaves behind a cavity with radius of $\sim\rho_{min}$ oscillating at plasma frequency

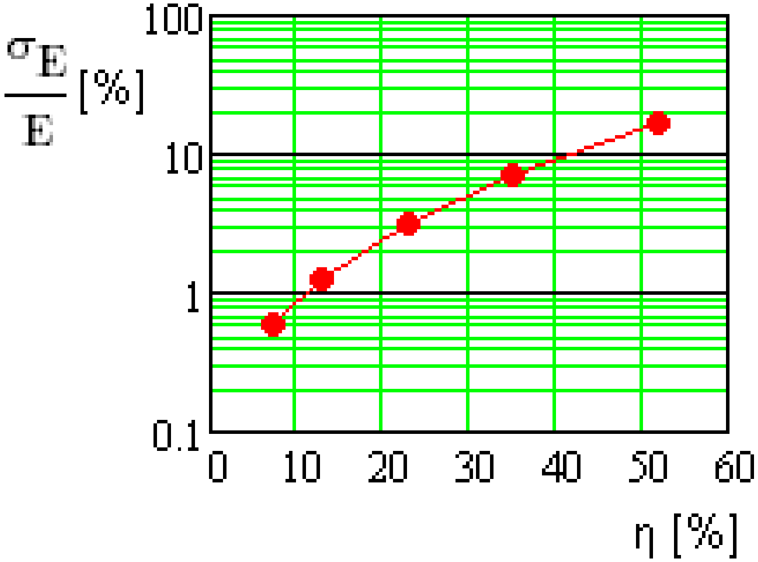


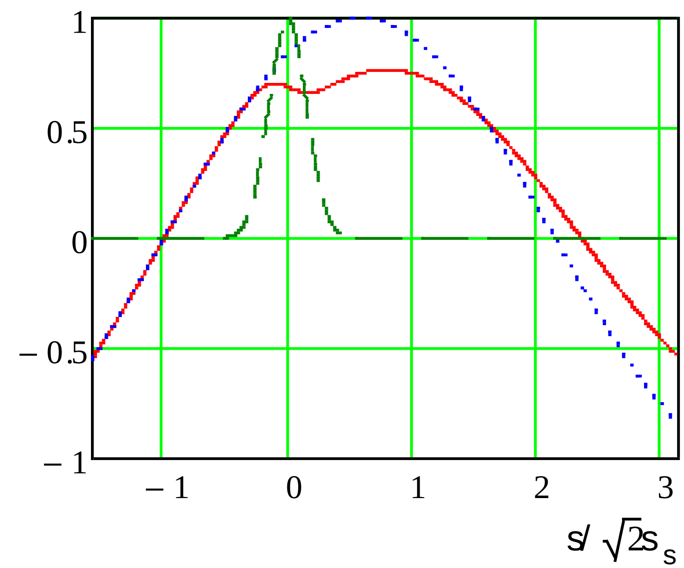
- With bunch intensity increase ρ_{min} achieves ρ_{max}. It corresponds to a transition to the blowout regime
  - Then, the deceleration achieves its maximum field of about

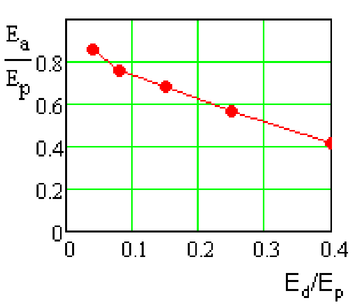
$$E_{\max} = \frac{4\pi e n_e}{k_p} = 30.2 \frac{\text{GV}}{\text{m}} \sqrt{\frac{n_e}{10^{17}\,\text{cm}^{-3}}}$$

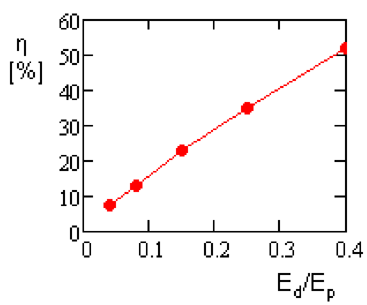
- A thin positron bunch ($\sigma_{\perp n} \le \rho_{min}$) collapses plasma electrons to its center. It makes plasma focusing very non-linear and results in drastic emittance increase for trailing particles

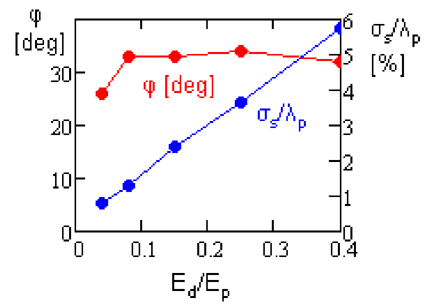
Energy Spread and Acceleration Efficiency of Gaussian Bunch











Longitudinal electric field with and without bunch field; $\Delta E/E_p$=0.15, ϕ=33^0,, σ_s/lp=0.024 => E_{acc}/E_p=0.68.

E_p – amplitude of plasma accelerating electric field

E_d – decelerating electric field in the bunch center

η - percentage of energy transferred from plasma to beam

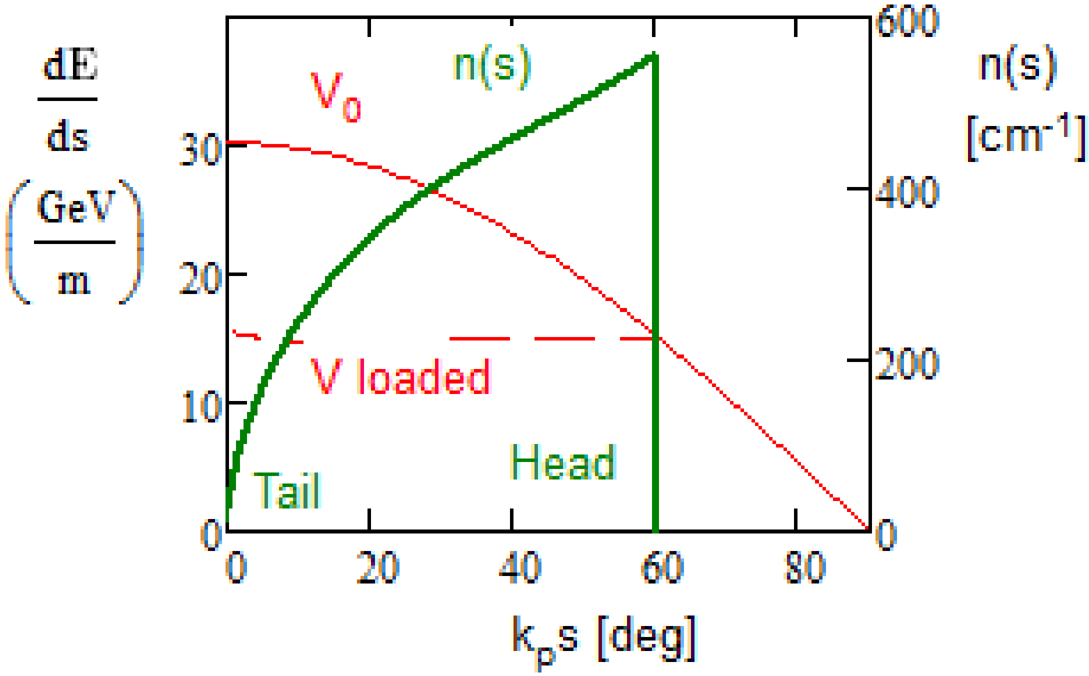
σ_E/E – rms energy spread in accelerated beam

E_a - average accelerating field

- Small energy spread is required to transfer the beam from one accelerating section to another and to focus the beam in IP
  - For 1% rms energy spread only ~9% of plasma energy can be transferred to the beam
    - ±2.5% total spread is a huge number

Bunch Shaping

- Shaping of bunch profile can significantly reduce accelerating voltage variations along the bunch
  - Growth of accelerating voltage is compensated by growth of decelerating force along the bunch



Longitudinal bunch density and loaded accelerating voltage for 50% beam loading

- The total bunch length is $\operatorname{acos}\left(V_{leaded}/V_0\right)$ (60 deg. for 50% loading)
- How such shapes can be created for the required bunch brightness is an open question

Plasma Focusing

- Using Maxwell equation one obtains that magnetic field is zero in the absence of external currents

$$\mathrm{rot}\,\mathbf{B} = \frac{4\pi\mathbf{j}}{c} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \quad \mathbf{j}_\omega = \frac{n_e e^2 \mathbf{E}_\omega}{i\omega m_e} \quad \Rightarrow \quad \mathrm{rot}\,\mathbf{B}_\omega = \frac{i\omega}{c}\left(1 - \frac{\omega_p{}^2}{\omega^2}\right)\mathbf{E}_\omega = 0 \quad \Rightarrow \quad \mathbf{B} = 0$$

- In the case of quasi-linear regime this statement is not perfectly correct. However electric field makes major part of focusing

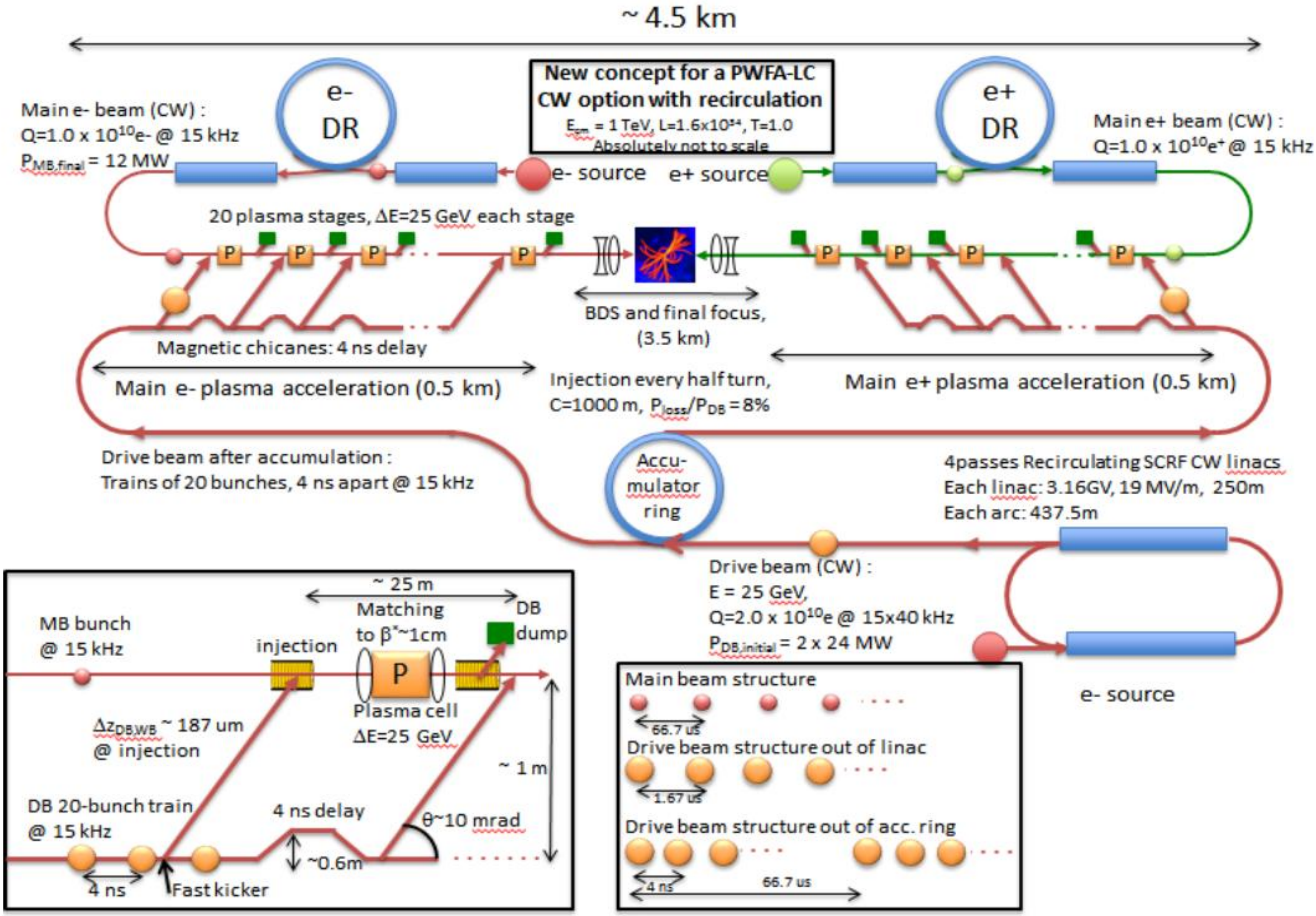
$$F_r = eE_r = -2\pi e^2 \delta n\, r - \frac{r}{2}\frac{\partial E_z}{\partial z}$$

- In the blowout regime focusing has contributions from E & B fields partially compensated. That leaves contribution from ions only

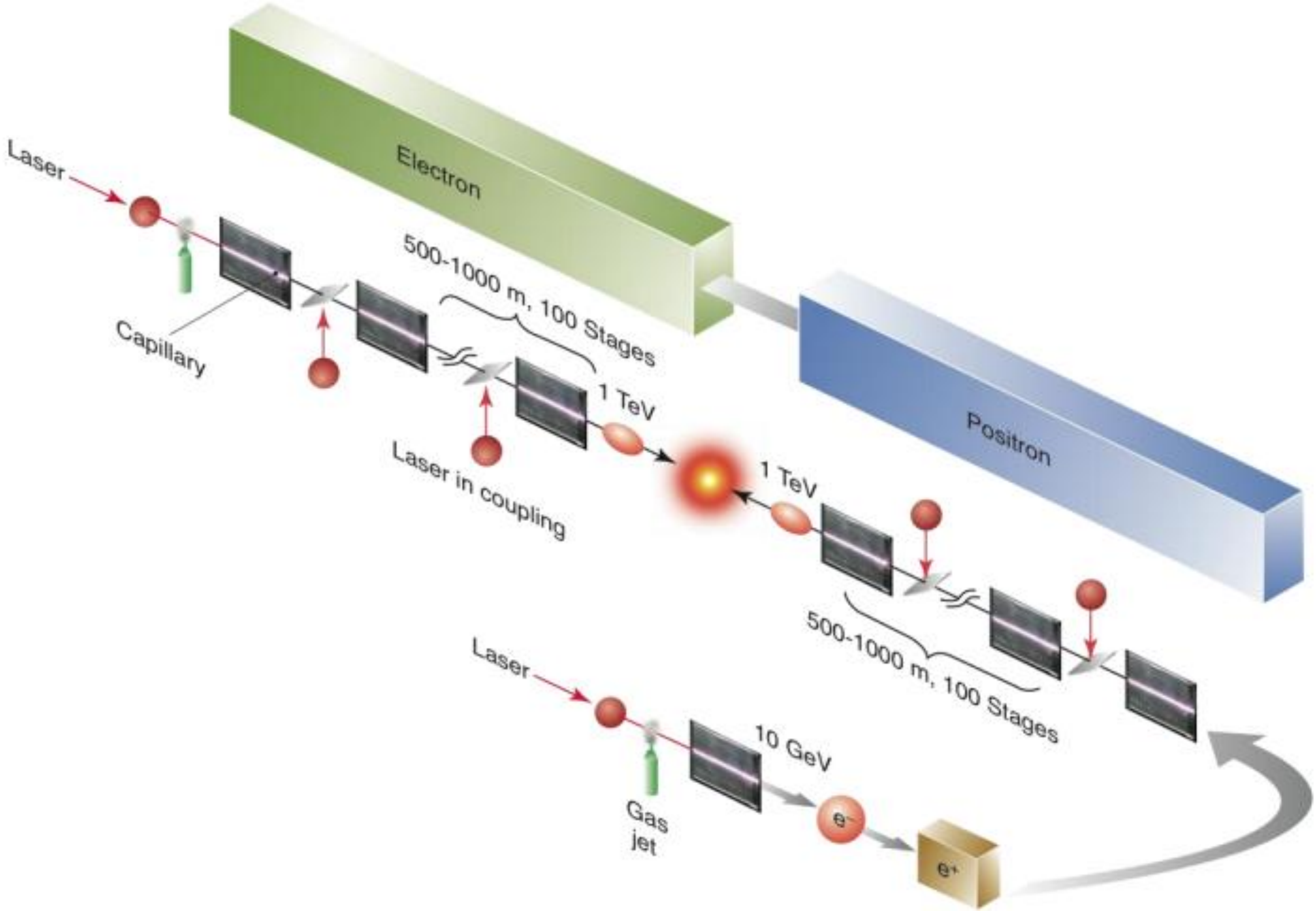
$$F_r = e\left(E_r + B_\theta\right) = -2\pi e^2\, n_i\, r$$

- Hollow plasma channel problem (suggested as a remedy from pinching)
  - An absence of charge near the axis removes its focusing
  - For laser driven plasma where an excitation wave propagates below c an absence of E&B compensation results in minor defocusing
  - Recent suggestion to use low density plasma in the channel requires more analysis

SLAC Proposal for e^+e^- Collider (Ref. [2])



LBNL Proposal



Picture is from [3]: Wim Leemans for the ICFA-ICUIL Joint Task Force, "White Paper of the ICFA-ICUIL Joint Task Force – High Power Laser Technology for Accelerators", ICFA BD (2011)

Main Features of SLAC& BNL Proposals

	LBNL [1,3]	SLAC [2]
Plasma parameters		
Regime of acceleration	Quasi-linear	Bubble
Excitation type	Laser beam	Electron beam
$\Delta n/n$	~ 0.3	1
n_e, cm^{-3}	10^{17}	$2\cdot10^{16}$
Wave length, $2\pi/k_p$, μm	105	234
E_{max}, GV/m	30	13.5
Loaded dE/dx, GeV/m	5	7.6
Beam parameters (colliding bunches)		
Final beam energy, GeV	500	
Luminosity (geometr.), $cm^{-2}s^{-1}$	$1.9\cdot10^{34}$	
Repetition rate, kHz	15	
Particles per bunch, $N_e=N_p$	$4\cdot10^9$	10^{10}
Rms. norm. emittance, $\varepsilon_y/\varepsilon_x$, nm	$100/100^{\dagger}$	$10^4/35$
Beta at IP, β_x^*/β_y^*, mm	$1^{\dagger}$	11/0.1
Rms bunch length, μm	**1 ?**	20

†Emittance is not presented in the most recent publication

Issues with the LBNL Proposal

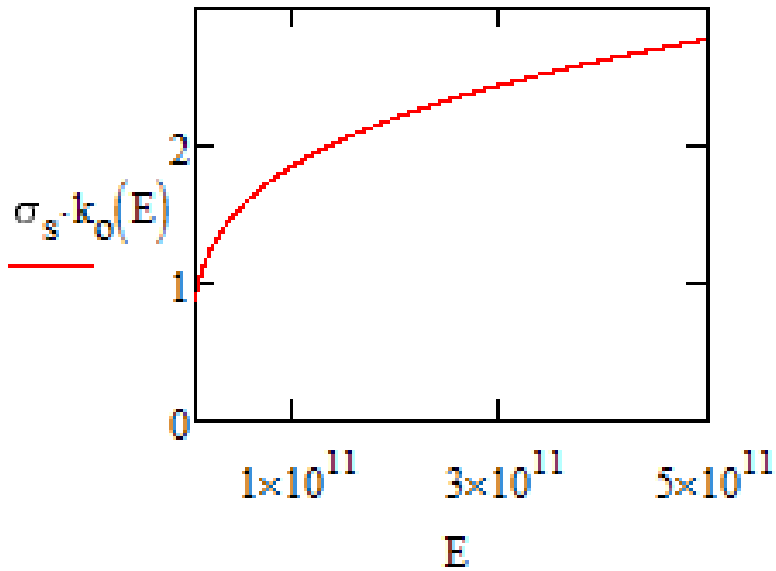
- RMS bunch length is only 3.4 deg. (1 μm)
  - It is incompatible with small energy spread at large beam loading
    - 50% beam loading => total bunch length of 60 deg. (~15-20 deg. rms)
- Bunch population is at the beam loading limit
    - Average deceleration force (~30GeV/m) exceeds accelerating gradient (~10 GeV/m)
- Pinching of plasma electrons by a positron bunch and their repulsion by an electron bunch destroys transverse focusing
  - An increase of transverse bunch size suggested as a remedy does not look as a possibility for the required small emittances
  - Large number of sections will require incredible control for the optics match between sections

Issues with the SLAC Proposal

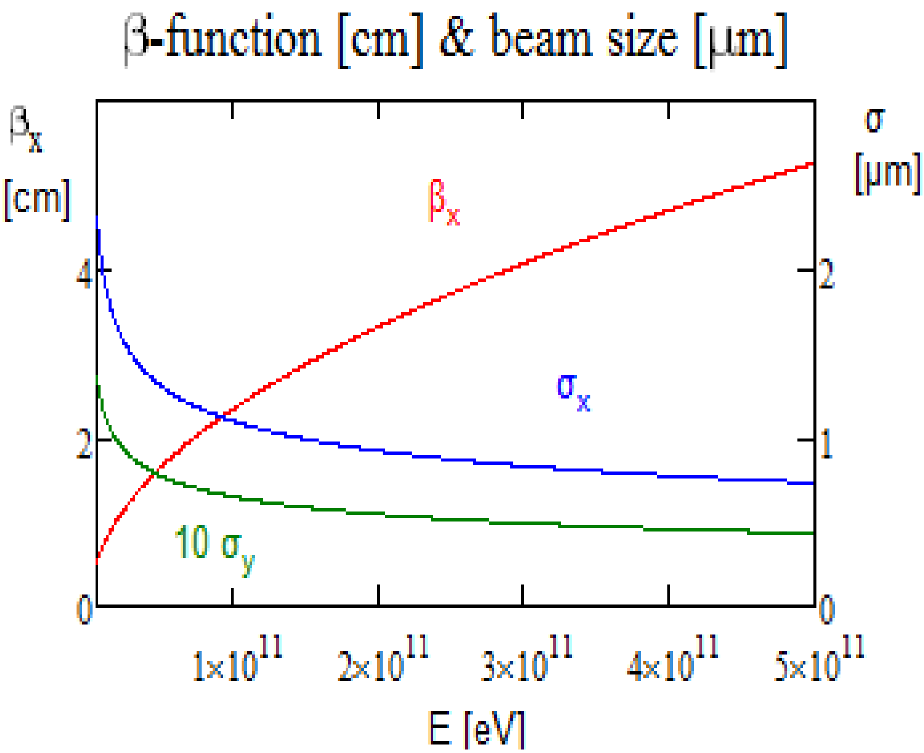
- Overall looks as a carefully thought-through proposal
- However:
  - There is no clear understanding of how positrons can be accelerated in the blowout regime
    - There are already a lot of publications and it is rather improbable that more work will open the way for acceleration of positron beam with required brightness
    - Presence of electrons on the axis required for transverse focusing will amplify the beam deceleration and pinching of electrons
  - Pinching of plasma protons completely destroys transverse focusing for electrons
    - Addressing ion collapse requires an increase of emittances or decrease of particle number in the bunch with consecutive decrease of luminosity

      ⇒ It creates a possibility for $\gamma-\gamma$ collider with compromised luminosity

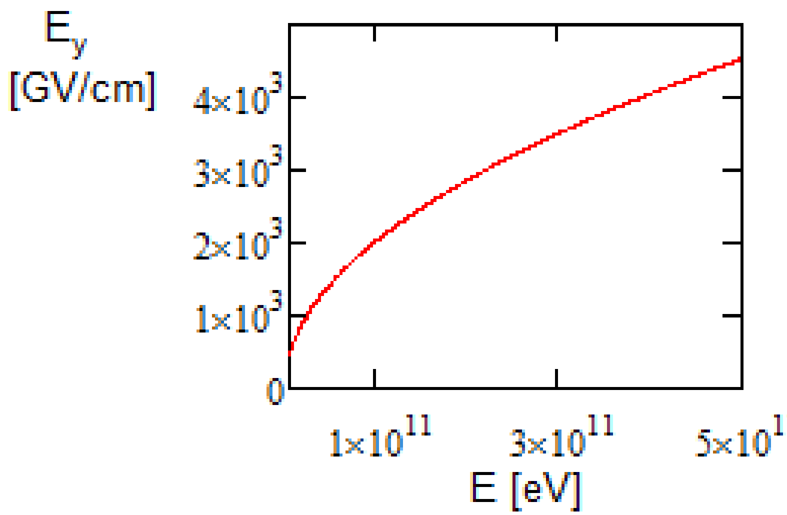
Issues with the SLAC Proposal (continue)



Phase advance of plasma ions oscillations in the field of electron bunch



- Impact ionization by bunch field is not negligible problem
  - Compare to el. field at a_0 in the hydrogen atom is ~6 GV/cm
- It prohibits a usage of heavy atoms in plasma



Conclusions

- Better understanding of proposals by all involved parties is desirable
  - Face-to-face discussion with SLAC and LBNL proponents of e+e- collider based on plasma acceleration should resolve some of these issues
- Beam interaction with plasma puts severe limitations on efficiency of the energy transfer from plasma to beam and the emittance conservation
  - As far as we can presently judge an acceleration of "a collider quality" beam required to achieve luminosity comparable to 10^{34} $cm^{-2}s^{-1}$ is not supported by the present proposals
- No doubts Plasma acceleration presents
  - interesting scientific subject
  - can find good application in a number of fields
  - But it hardly can be a valuable tool for future e^+e^- colliders of TeV energy scale

References

[1] C. B. Schroeder, E. Esarey, C. G. R. Geddes, C. Benedetti, and W. P. Leemans, "Physics considerations for laser-plasma linear colliders", PRST-AB, 13, 101301 (2010)

[2] E. Adli, J.P.Delahaye, S.J.Gessner, M.J. Hogan, T. Raubenheimer (SLAC), W.An, C. Joshi, W.Mori (UCLA) "A Beam Driven Plasma-Wakefield Linear Collider: From Higgs Factory to Multi-TeV", SLAC Publication SLAC-PUB-1542, April 10, 2013.

[3] Wim Leemans for the ICFA-ICUIL Joint Task Force, "White Paper of the ICFA-ICUIL Joint Task Force – High Power Laser Technology for Accelerators", ICFA BD Newsletter #56 http://icfa-usa.jlab.org/archive/newsletter.shtml

Backup slides

Transverse Wake

- There is no transverse wake in uniform plasma
  - However focusing of trailing particles do exist (detuning wake)
- Beam acceleration perturbs plasma density and creates accelerating channel and, consequently, transverse wake
- For small beam size ($\sigma_{b\perp} \ll c/\omega_p$) the wake field is nearly uniform in transverse plane
  - The wake-function grows almost linearly
  - In logarithmic approximation it is

$$W_\perp = 2\left(\frac{\omega_p}{c\,\sigma_\perp}\right)^2 \left(\frac{\Delta n}{n}\right)_e (s-s')\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \xrightarrow{c/\omega_p=\sigma_\perp} 2\left(\frac{\Delta n}{n}\right)_e \frac{s-s'}{\sigma_\perp{}^4}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

- Comparing it with focusing strength of plasma channel we obtain at the bunch end

$$\frac{E_{wake}(s=L_b)}{E_{plasma\_foc}} \approx \frac{2L_b}{\sigma_\perp}\frac{(dE/ds)_{loss}}{(dE/ds)_{\max}}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

where $(dE/ds)_{loss}$ - average energy loss in plasma
$(dE/ds)_{max}$ - maximum accelerating field for given plasma density

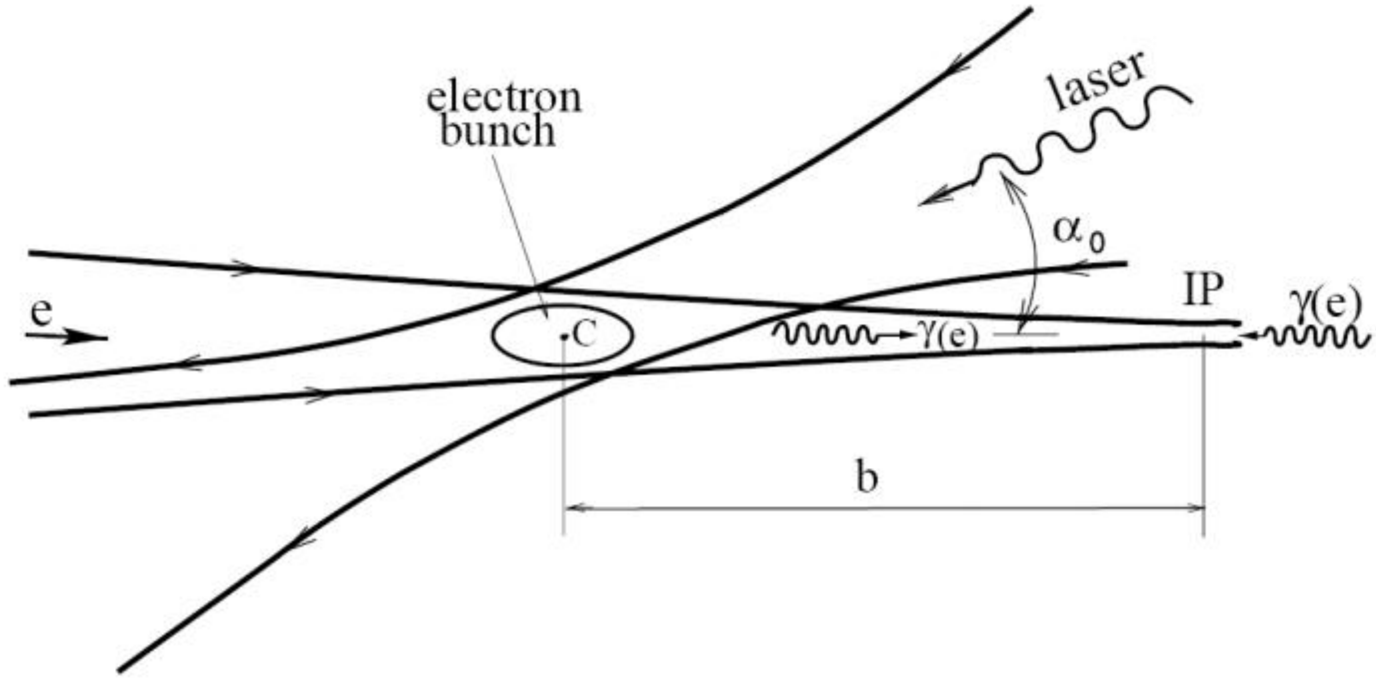


Illustration of principle of a $\gamma-\gamma$ collider